

# Guess paper Annual 2022



پنجاب کے تمام بورڈ کے لیے

# کام سالی کا تھوڑہ

صرف پندرہ دن کے اندر بورڈ امتحان کی مکمل تیاری کریں

# Mathematics

For Inter Part - I

اب فیل ہونا بھول جائیں

☆ پپر Setter کے ذہن کو مد نظر رکھ کر تیار کیے گئے سوالات

☆ یاد رکھیں! اب وقت انتہائی کم رہ گیا ہے۔

☆ صرف پندرہ دن کے اندر بورڈ امتحان کی مکمل تیاری کریں

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## QUESTION NO. 2

1)	Prove the rules of addition. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
2)	Prove the rules of addition. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
3)	Prove that $-\frac{7}{12} - \frac{5}{18} = \frac{21-10}{36}$
4)	Find the sum, difference and product of the complex numbers (8,9) and (5, -6)
5)	Simplify: $(-1)^{\frac{-21}{2}}$
6)	Simplify (2,6)(3,7)
7)	Simplify $(2,6) \div (3,7)$ Hint: $\frac{(2,6)}{(3,7)} = \frac{2+6i}{3+7i} \times \frac{3-7i}{3-7i}$ etc.
8)	Simplify $(5, -4) \div (-3, -8)$
9)	Find the multiplicative inverse of the numbers: (-4,7)
10)	Find the multiplicative inverse of the numbers: $(\sqrt{2}, -\sqrt{5})$
11)	Factorize: $9a^2 + 16b^2$
12)	Factorize: $3x^2 + 3y^2$
13)	Separate into real and imaginary parts (write as a simple complex number) $\frac{2-7i}{4+5i}$
14)	Find the multiplicative Inverse of each of the numbers. (1,2)
15)	Prove that $\bar{z} = z$ if $z$ is real.
16)	Simplify by expressing in the from $a + bi$ $(2 + \sqrt{-3})(3 + \sqrt{-3})$
17)	Show that $\forall z \in C. z^2 + \bar{z}^2$ is a real number
18)	Show that $\forall z \in C. (z - \bar{z})^2$ is a real number
19)	Simplify: $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$
20)	Find moduli of the complex numbers: $1 - i\sqrt{3}$
21)	Write two proper subsets of the set: $\{a, b, c\}$
22)	Write down the power set of the each of the sets: $\{+, -, \div, \times\}$
23)	Write the converse, inverse and contrapositive of the conditionals: $\sim p \rightarrow q$
24)	Find $x$ and $y$ if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$
25)	xiv  If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ , find the values of $a$ and $b$ .
26)	if $A$ and $B$ are square matrices of the same order, then explain why in general: $(A + B)(A - B) \neq A^2 - B^2$
27)	solve the equation $\begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$
28)	without expansion show that $\begin{bmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix} = 0$
29)	Without expansion show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

30) Show that  $\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix} = 0$

31) Without expansion verify that  $\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ , & & c \end{vmatrix}$

32) Show that  $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$

33) Salve the equation by factorization:  $\frac{a}{ax-1} + \frac{b}{bx-1} = a+b; x \neq \frac{1}{a}, \frac{1}{b}$

34) Show that:  $x^3 - y^3 = (x-y)(x-\omega y)(x-\omega^2 y)$

35) Evaluate:  $(1+\omega-\omega^2)(1-\omega+\omega^2)$

36) Evaluate:  $(1+\omega-\omega^2)^8$

37) Evaluate:  $(-1 + \sqrt{-3})^5 + (-1 - \sqrt{3})^5$

38) Solve the equations:  $2x^4 - 32 = 0$

39) Salve the equations:  $x^3 + x^2 + x + 1 = 0$

40) Use the factars theorem ta determine if the first polynomial is a factor of the second polynomial.  $\omega + 2, 2\omega^3 + \omega^2 - 4\omega + 7$

41) Find four fourth roots of 16

42) If w is a cube root of unity, form an equation whose roots are  $2\omega$  and  $2\omega^2$

43) Find roots of the equation by using quadratic formula:  $15x^2 + 2ax - a^2 = 0 \quad \frac{1}{a^2} + \frac{1}{\beta^2}$

44) If  $\alpha, \beta$  are the raots of  $3x^2 - 2x + 4 = 0$ , find the values of  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

45) If  $\alpha, \beta$  are the raots of  $3x^2 - 2x + 4 = 0$ , find the values of  $\alpha^2 - \beta^2$

46) if  $a, \beta$  are the roots of  $x^2 - px - p - c = 0$ , prove that  $(1+d)(1+\beta) = 1 - c$

47) if  $a, \beta$  are the roots of the equation  $ax^2 + bx + c = 0$ , from the equation whose raots are.  $\alpha^2, \beta^2$

48) If  $\alpha, \beta$  are the raots of the equation  $ax^2 + bx + c = 0$ , fram the equation whose r0ots are.  $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$

49) Discuss the nature of the raots of the equation.  $x^2 - 5x + 6 = 0$

50) Discuss the nature of the roots of the equation.  $25x^2 - 30x + 9 = 0 \quad x^2 - 2\left(m + \frac{1}{m}\right)x + 3 ; m \neq 0$

51) Show that the roots of the equation will be rational:  $(p+q)x^2 - px - q = 0$

52) Find two consecutive numbers, whose productis 132 . (Hint: Suppose the numbers are  $x$  and  $x + 1$ )

53) Use synthetic division to find the quotient and the remainder when the polynomial  $x^4 - 10x^2 - 2x + 4$  is divided by  $x + 3$

54) Discuss the nature of the raots of the equations:  $2x^2 + 5x - 1 = 0$

55) Which of the following sets have closure property w.r.t addition and multiplication ? (i){0, -1} (ii){1, -1}

56) Theorems:  $\forall z, z_1, z_2 \in C z\bar{z} = |z|^2$

57) Theorems  $\forall z, z_1, z_2 \in C \left(\frac{z_1}{z_2}\right) = \frac{\overline{z_1}}{\overline{z_2}}$

58) Find the power set.  $\{\{a, b\}, \{b, c\}, \{d, e\}\}$

59) Reversal law of inverse if a, b are elements of group G, then show that  $(ab)^{-1}b^{-1}a^{-1}$

- 60) Find  $x$  and  $y$  if  $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$
- 61) Solve the equation  $x^{\frac{1}{2}} - x^{\frac{1}{4}} - 6 = 0$
- 62) Solve the equation  $x^{\frac{2}{5}} + 8 = 6x^{\frac{1}{5}}$
- 63) Prove Three Cube Roots of Unity .
- 64) The Sum of all the three cube roots of unity is zero. i.e.,  $1 + \omega + \omega^2 = 0$

# QUESTION NO. 3

- 65) Resolve the following into Partial Fraction:  $\frac{6x^3+5x^2-7}{2x^2-x-1}$
- 66) Resolve the Partial Fraction:  $\frac{9}{(x+2)^2(x-1)}$
- 67) Resolve,  $\frac{7x+25}{(x+3)(x+4)}$  into Partial Fractions.
- 68) Write the first four terms of the sequences, if  $a_n = (-1)^n(2n - 3)$
- 69) Write the first four terms of the sequences, if  $a_n = na_{n-1}, a_1 = 1$
- 70) Find the indicated term of the sequence:  $1-3, 5, -7, 9, -11, \dots, a_8$
- 71) Find the 13 th term of the sequence  $x, 1, 2 - x, 3 - 2x, \dots$
- 72) Which term of the A.P.  $-2, 4, 10, \dots$  is 148?
- 73) Resolve the Partial Fraction:  $\frac{x^2}{(x-2)(x-1)^2}$
- 74) Resolve,  $\frac{x^2+x-1}{(x+2)^3}$  into Partial Fractions.
- 75) Write the first four terms of the sequences, if  $a_n = (-1)^n n^2$
- 76) Write the first four terms of the sequences, if  $a_n = \frac{n}{2n+1}$
- 77) Write the first four terms of the sequences, if  $a_n - a_{n-1} = n + 2, a_1 = 2$
- 78) If  $a_{n-3} = 3n - 5$ , find the nth term of the sequence.
- 79) Which term of the A.P.  $5, 2, -1, \dots$  is  $-85$  ?
- 80) How many terms are there in the A.P. in which  $a_1 = 11, a_n = 68, d = 3$ ?
- 81) If the nth term of the A.P. is  $3n - 1$ , find the A.P
- 82) xxvii) If  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in A.P., show that  $b = \frac{2ac}{a+c}$
- 83) Sum the series  $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \dots + a_{13}$
- 84) Sum the series  $1 + 4 - 7 + 10 + 13 - 16 + 19 22 - 25 + \dots \dots$  to 3 n terms.
- 85) Find the 11 th term of the sequence,  $1+i, 2, \frac{4}{1+i}$
- 86) Find G.M. between  $-2$  and  $8$
- 87) For what value of  $n$ ,  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  is the positive geometric mean between  $a$  and  $b$
- 88) Find the 9th term of the harmonic sequence  $\frac{-1}{5}, \frac{-1}{3}, -1, \dots$
- 89) Iv) If  $5$  is the harmonic mean between  $2$  and  $b$ , findxxvi) Find the nth term of the sequence,  $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2,$
- 90) Find A · M. between  $x - 3$  and  $x + 5$

- 91) Find three A. Ms between 3 and 11 .
- 92) Sum the series  $1.11 + 1.41 + 1.71 + \dots + a_{10}$
- 93) How many terms of the series  $-7 + (-5) + (-3) + \dots$  amount to 65?
- 94) Find the 12 th term of  $1 + 1, 2i, -2 + 2i, \dots$
- 95) Find G.M. between  $-2i$  and  $8i$
- 96) Insert two G.Ms. between 1 and 16
- 97) Find the 9th term of the harmonic sequence  $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$
- 98) The first term of an H.P. is  $-\frac{1}{3}$  and the fifth term is  $\frac{1}{5}$ . Find its 9th term.
- 99) If  $A, G$  and  $H$  are the arithmetic, geometric and harmonic means between  $a$  and  $b$  respectively, show that  $G^2 = AH$
- 100) Find  $A, G, H$  and show that  $G^2 = A \cdot H$ . if (i)  $a = -2, b = 8$  (ii)  $a = 2i, b = 4i$  (iii)  $a = 9, b = 4$
- 101) Find the sequence if  $a_n - a_{n-1} = n + 1$  and  $a_4 = 14$
- 102) If  $a_{n-2} = 3n - 11$ , find the nth term of the sequence.
- 103) Find the sum of the infinite G.P.  $2, \sqrt{2}, 1, \dots$
- 104) Write in the factorial form:  $n(n-1)(n-2) \dots (n-r+1)$
- 105) Write in the factorial form:  $\frac{(n+1)(n)(n-1)}{3 \cdot 2 \cdot 1}$
- 106) Find the value of  $n$  when:  ${}^n P_2 = 30$ ,
- 107) Find the value of  $n$  when:  ${}^n P_4 : {}^{n-1} P_3 = 9 : 1$
- 108) How many arrangements of the letters of the words, taken all together, can be made: i) PAKPATTAN ii) PAKISTAN
- 109) In how many ways can 4 keys be arranged on a circular key ring?
- 110) Find  $A, G, H$  and verify that  $A > G > H$  ( $G > 0$ ), if (i)  $a = 2, b = 8$  (ii)  $a = \frac{2}{5}, b = \frac{8}{5}$
- 111) Find the number of terms in the A.P. if:  $a_1 = 3, d = 47$  and  $a_n = 59$
- 112) Find three A.Ms between  $\sqrt{2}$  and  $3\sqrt{2}$ .
- 113) Find the nth and 8 th terms of H.P :  $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$
- 114) Evaluate:  $\frac{4!2!}{15!(15-15)}$
- 115) Write in the factorial form:  $(n+2)(n+1)(n)$
- 116) Find the value of  $n$  when:  ${}^{11} P_n = 11 \cdot 10 \cdot 9$
- 117) How many signals can be given by 5 flags of different colours, using 3 flags at a time?
- 118) How many arrangements of the letters of the words, taken all together, can be made: i) MATHEMATICS ii) ASSASSINATION
- 119) How many necklaces can be made from 6 beads of different colours?
- 120) Evaluate:  ${}^n C_4$
- 121) Find the value of  $n$ , when  ${}^n C_{10} = \frac{12 \times 11}{2!}$
- 122) Find the value of  $n$  and  $r$ , when  ${}^n C_r = 35$  and  ${}^n P_r = 210$
- 123) Experiment: A die is rolled. The top shows Events Happening: (i) 3 or 4 dots (ii) dots less than 5.
- 124) Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?
- 125) If  ${}^n C_8 = {}^n C_{12}$ , find  $n$ .
- 126) A die is rolled. What is the probability that the dots on the top greater than 4?
- 127) Using the binomial theorem expand:  $(a + 2b)^5$
- 128) Find the value of  $n$ , when  ${}^n C_5 = {}^n C_4$

- 129) Find the value of  $n$ , when  ${}^nC_{12} = {}^nC_6$
- 130) What is the probability that a slip of numbers divisible by 4 is picked from the slips bearing number 1,2,3, ...,10?
- 131) Using binomial theorem, find the values to three places of decimals  $\sqrt{99}$
- 132) Evaluate  $\sqrt[8]{30}$  correct to three places of decimal.
- 133) If  $y = 1 + 2x + 4x^2 + 8x^3 + \dots$  Show that  $x = \frac{y-1}{2y}$
- 134) Expand up to four terms taking the values of  $x$  such that the expansion is valid.  $(1-x)^{\frac{1}{2}}$
- 135) Using binomial theorem, find the values to three places of decimals  $(1.03)^{\frac{1}{2}}$
- 136) If  $a, b, c, d$  are in G.P, prove that  $a^2 - b^2, b^2 - c^2, c^2 - d^2$  are in G.P

## QUESTION NO. 4

- 137) Express the sexagesimal measures of angles in radians:  $75^\circ 6' 30''$
- 138) Convert the following radian measures of angles into the measures of sexagesimal system:  $\frac{2\pi}{3}$
- 139) Find I, when:  $\theta = 65^\circ 20'$ ,  $r = 18 \text{ mm}$
- 140) Verify:  $\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1 : 2 : 3 : 4$ .
- 141) What is the circular measure of the angle between the hands of a watch at 4 O'clock?
- 142) Find I, when:  $\theta = \pi$  radians,  $r = 6 \text{ cm}$
- 143) Find  $r$ , when:  $I = 5 \text{ cm}$ ,  $\theta = \frac{1}{2}$  radian
- 144) Verify:  $60\cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$
- 145) Verify:  $\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{3} : \tan^2 \frac{\pi}{4} = 2 : 3 : 1$  Verify:  $2\sin 45^\circ + \frac{1}{2} \cosec 45^\circ = \frac{3}{\sqrt{2}}$
- 146) Evaluate:  $\frac{1-\tan^2 \frac{\pi}{3}}{1+\tan^2 \frac{\pi}{3}}$
- 147) Verify the following when  $\theta = 30^\circ, 45^\circ$   $\cos 2\theta = 2\cos^2 \theta - 1$
- 148) Prove the following identities, state the domain of  $\theta$  in each case:  $\sec^2 \theta - \cosec^2 \theta = \tan^2 \theta - \cot^2 \theta$
- 149) Prove the following identities, state the domain of  $\theta$  in each case:  $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$
- 150) Prove the following identities, state the domain of  $\theta$  in each case:  $\cos^2 \theta - \sin^2 \theta = \frac{1-\tan^2 \theta}{1+\tan^2 \theta}$
- 151) Prove the following identities, state the domain of  $\theta$  in each case:  $\frac{\sin \theta}{1+\cos \theta} + \cot \theta = \cosec \theta$
- 152) Prove the following identities, state the domain of  $\theta$  in each case:  $\frac{\cot^2 \theta - 1}{1+\cot^2 \theta} = 2\cos^2 \theta - 1$
- 153) xlvi) Prove that  $\cos^2 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$ ,
- 154) Verify the following when  $\theta = 30^\circ, 45^\circ$   $\sin 2\theta = 2\sin \theta \cos \theta$
- 155) Find  $x$ , if  $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$
- 156) Prove the following identities, state the domain of  $\theta$  in each case:  $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{\cot \theta - 1}{\cot \theta + 1}$
- 157) Prove the following identities, state the domain of  $\theta$  in each case:  $\frac{1+\cos \theta}{1-\cos \theta} = (\cosec \theta + \cot \theta)^2$
- 158) Prove that:  $\sqrt{\frac{1-\sin \theta}{1+\sin \theta}} = \sec \theta - \tan \theta$ , where
- 159) Show that:  $\cot^4 \theta + \cot^2 \theta = \cosec^4 \theta - \cosec^2 \theta$  where  $\theta$  is not an integral multiple of  $\frac{\pi}{2}$

- 160) Prove:  $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$
- 161) Prove that:  $\tan(45^\circ + A)\tan(45^\circ + A) = 1$
- 162) Prove that:  $\frac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi} = \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$
- 163) Prove that:  $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$
- 164) Prove the identities:  $\cot\alpha - \tan\alpha = 2\cot 2\alpha$
- 165) Prove the identities:  $1 + \tan\alpha \tan 2\alpha = \sec 2\alpha$
- 166) Express the products as sums or differences:  $2\sin 3\theta \cos \theta$
- 167) Prove:  $\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha \cos\alpha$
- 168) If  $\alpha, \beta, \gamma$  are the angles of a triangle  $ABC$ , then prove that:  $\cos(\alpha + \beta) = \cos\gamma$
- 169) Find the values:  $\sin 105^\circ$  Ixviii) Prove that:  $\cos(\alpha + 45^\circ) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$
- 170) Prove that:  $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right) = \cos 0$
- 171) Show that:  $\cot(\alpha + \beta) = \frac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta}$
- 172) Find the values of  $\sin 2\alpha, \cos 2\alpha$  and  $\tan 2\alpha$ , when:  $\cos\alpha = \frac{3}{5}$ , where  $0 < \alpha < \frac{\pi}{2}$
- 173) Prove the identities:  $\frac{\sin 2\alpha}{1 + \cos 2\alpha} = \tan\alpha$
- 174) Prove the identities:  $\frac{2\sin\theta \sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta \tan\theta$
- 175) Express the sums or differences as products:  $\sin(x + 30^\circ) + \sin(x - 30^\circ)$
- 176) Express  $\sin 5x + \sin 7x$  as a product.
- 177) Find the periods of the functions:  $\tan 4x$
- 178) Find the periods of the functions:  $\sin\frac{x}{5}$
- 179) Solve the right triangle  $ABC$ , in which  $\gamma = 90^\circ, a = 3.28, b = 5.74$
- 180) At the top of a cliff 80 m high, the angle of depression of a boat is  $12^\circ$ . How far is the boat from the cliff?
- 181) When the angle between the ground and the sun is  $30^\circ$ , flag pole casts a shadow of 40 m long. Find the height of the flag.
- 182) Solve the triangle  $ABC$  in which:  $a = \sqrt{3} - 1, b = \sqrt{3} + 1$  and  $\gamma = 60^\circ$
- 183) Solve the triangle  $ABC$  in which:  $a = 7, b = 3$  and  $\gamma = 38^\circ 13'$
- 184) Prove that:  $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$
- 185) Find the periods of the functions:  $\cos 2x$
- 186) Solve the triangles, in which:  $a = 32, b = 40, c = 66$
- 187) Find the smallest angle of the triangle  $ABC$ , When  $a = 37.34, b = 3.24, c = 35.06$
- 188) Find the area of the triangle  $ABC$ , given two sides and their included angle:  $b = 37, c = 45, \alpha = 30^\circ 50'$
- 189) Show that:  $r_2 = 4R \cos \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\gamma}{2}$
- 190) Prove that:  $rr_1r_2r_3 = \Delta^2$
- 191) Find  $R, r, r_1, r_2$  and  $r_3$ , if measures of the sides of triangle  $ABC$  are  $a = 34, b = 20, c = 42$
- 192) Prove that in an equilateral triangle.  $r: R : r_1:r_2:r_3 = 1:2:3:3:3$
- 193) Prove that:  $(r_1 + r_2)\tan\frac{\gamma}{2} = c$ .
- 194) Evaluate without using tables / calculator:  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

- 197) Without using table / Calculator show that:  $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$
- 198) Find the area of the triangle ABC, given one side and two angles:  $b = 25.4$ ,  $\gamma = 36^\circ 41'$ ,  $\alpha = 45^\circ 17'$
- 199) Find the area of the triangle ABC, given three sides:  $a = 18$ ,  $b = 24$ ,  $c = 30$
- 200) Show that:  $r_3 = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$
- 201) Prove that:  $r_1r_2r_3 = rs^2$
- 202) Prove that in an equilateral triangle. r: R:  $r_1 = 1: 2: 3$
- 203) Prove that:  $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$
- 204) Prove that:  $(r_3 - r)\cot\frac{\gamma}{2} = c$ .
- 205) Without using table / Calculator show that:  $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$
- 206) Find the value of each expression:  $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$
- 207) Show that:  $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$
- 208) Show that:  $\cos^{-1}(-x) = \pi - \cos^{-1}x$
- 209) Solve the trigonometric equations:  $\sec^2\theta = \frac{4}{3}$
- 210) Find the values of  $\theta$  satisfying the equations:  $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$
- 211) Solve:  $\sin x + \cos x = 0$
- 212) Prove that  $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$
- 213) Show that:  $\sin^{-1}(-x) = -\sin^{-1}x$
- 214) Show that:  $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$
- 215) Show that  $\cos^{-1}\frac{12}{13} = \sin^{-1}\frac{5}{13}$
- 216) Prove that  $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$
- 217) Prove that : .  $R = \frac{abc}{4\Delta}$

## LONG Q.NO. 5

- 1) Reversal law of inverse if a ,b are elements of group G , then show that  $(ab)^{-1} = b^{-1}a^{-1}$
- 2) If G is a group under the operation  $\otimes$  and  $a,b \in G$  , find the solutions of the equation.  $a * x = b$  ,  $x * a = b$
- 3) Show that the set  $\{1, \omega, \omega^2\}$ , when  $\omega^3 = 1$ , is an Abelian group w.r.t. ordinary multiplication.
- 4) Show that  $\begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix} = l^2(3a+l)$
- 5) Use Crammer's rule to solve the system.  $\begin{cases} 3x_1 + x_2 - x_3 = -4 \\ x_1 + x_2 - 2x_3 = -4 \\ -x_1 + 2x_2 - x_3 = 1 \end{cases}$
- 6) Show that  $\begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & c & c+\lambda \end{vmatrix} = \lambda^2(a+b+c)$
- 7) Solve the equation by Cramer's rule.  $\begin{cases} 2x_1 + 2x_2 + x_3 = 8 \\ x_1 + 2x_2 + 2x_3 = 6 \\ x_1 - 2x_2 - x_3 = 1 \end{cases}$
- 8) Find the value of  $\lambda$  for the system of non-trivial solution . Also solve the system for the value of  $\lambda$   

$$\begin{cases} x_1 + 4x_2 + \lambda x_3 = 0 \\ 2x_1 + x_2 - 3x_3 = 0 \\ 3x_1 + \lambda x_2 - 4x_3 = 0 \end{cases}$$

9) Solve the system of linear equation by Cramer's rule  $\begin{cases} 2x_1 + 2x_2 + x_3 = 8 \\ x_1 + 2x_2 + 2x_3 = 6 \\ x_1 - 2x_2 - x_3 = 1 \end{cases}$

10) Use matrices to solve the system  $\begin{cases} 2x_1 + x_2 + 3x_3 = 3 \\ x_1 + x_2 - 2x_3 = 0 \\ -3x_1 - x_2 + 2x_3 = -4 \end{cases}$

## LONG Q.NO. 6

11) Solve the equation  $(x + 1)(2x + 3)(2x + 5)(x + 3) = 945$

12) Solve the equation  $4.2^{2x+1} - 9.2^x + 1 = 0$

13) Solve the equation  $3^{2x-1} - 12.3^x + 81 = 0$

14) Solve the equation  $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$

15) Solve the equation:  $(x + 4)(x + 1) = \sqrt{x^2 + 2x - 15} + 3x + 31$

16) Prove that complex cube roots of  $-1$  are  $\frac{1+\sqrt{-3}i}{2}$  and  $\frac{1-\sqrt{-3}i}{2}$ ; and hence prove that  $\left(\frac{1+\sqrt{-3}i}{2}\right)^9 + \left(\frac{1-\sqrt{-3}i}{2}\right)^9 = -2$

17) If the roots of  $px^2 + qx + q = 0$  are  $\alpha$  and  $\beta$  then prove that  $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$  Show that the roots of  $x^2 + (mx + c)^2 = a^2$  will be equal, if  $c^2 = a^2(1 + m^2)$  xoxix) Prove that  $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$  will have equal roots, if  $c^2 = a^2m^2 + b^2$ ;  $a \neq 0, b \neq 0$

18) Resolve the Partial Fraction:  $\frac{x-1}{(x-2)(x+1)^3}$

19) Resolve the Partial Fraction:  $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$

20) Resolve the Partial Fraction:  $\frac{9x-7}{(x^2+1)(x+3)}$

21) xlvi ) Resolve the Partial Fraction:  $\frac{x^2+1}{x^3+1}$

22) xlvi) Resolve the Partial Fraction:  $\frac{1}{(x-1)^2(x^2+2)}$

23) Resolve,  $\frac{1}{(x+1)^2(x^2-1)}$  into Partial Fractions.

24) Resolve,  $\frac{4x^2}{(x^2+1)^2(x-1)}$  into Partial Fractions.

## LONG Q.NO. 7

25) Find  $n$  so that  $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$  may be the A.M. between  $a$  and  $b$ .

26) The sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers.

27) Find four numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

28) Find the five numbers in A.P. whose sum is 25 and the sum of whose squares is 135.

29) If  $\frac{1}{a}, \frac{1}{b}$  and  $\frac{1}{c}$  are in G.P. show that the common ratio is  $\pm \sqrt{\frac{a}{c}}$

30) If there consecutive numbers in A.P. are increased by 1, 4, 15 respectively, the resulting numbers are in G.P.

31) Find the original numbers if their sum is 6 .

32) Sum the series  $2 + (1 - i) + \left(\frac{1}{i}\right) + \dots$  to 8 terms.

33) If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \dots$ and if $0 < x < 2$ , then prove that $x = \frac{2y}{1+y}$
34) If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \dots$ and if $0 < x < -\frac{3}{2}$ then show that $x = \frac{3y}{2(1+y)}$
35) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k.
36) Find n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be H.M. between a and b
37) If the H.M and A.M between two numbers are 4 and $\frac{9}{2}$ respectively, find the numbers.
38) If the positive G.M. and H.M. between two numbers are 4 and $\frac{16}{5}$
39) Use mathematical induction to prove the formula for every positive integer n. $1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n-1)}{2}$
40) Find the term involving $x^4$ in the expansion of $(3 - 2x)^7$
41) Find the coefficient of $x^5$ in the expansion of $(x^2 - \frac{3}{2x})^{10}$
42) Find the term independent of x in the expansion: $(x - \frac{2}{x})^{10}$ xaxiv) Determine the middle term or terms in the expansions: $(\frac{1}{x} - \frac{x^2}{2})^{12}$ xoxy) Determine the middle term or terms in the expansions: $(\frac{3}{2}x - \frac{1}{3x})^{11}$
43) Find the coefficient of $x^n$ in the expansion. $\frac{(1+x)^2}{(1-x)^2}$
44) Identify the series $1 - \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1.3}{2!4}\left(\frac{1}{4}\right)^2 - \frac{1.3.5}{3!8}\left(\frac{1}{4}\right)^3 + \dots$ as binomial expansion and find its sum.
45) Identify the series $1 - \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{2}\right)^3 + \dots$ as binomial expansion and find its sum.
46) If $y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \dots$ , Prove that $y^2 + 2y - 2 = 0$
47) If $2y = \frac{1}{2^2} + \frac{1.3}{2!} \cdot \frac{1}{2^4} + \frac{1.3.5}{3!} \cdot \frac{1}{2^6} + \dots$ prove that $4y^2 + 4y - 1 = 0$

## LONG Q.NO. 8

48) Find the values of the remaining trigonometric functions: $\cos \theta = \frac{9}{41}$ and the terminal arm of the angle is in quad. IV.
49) If $\operatorname{cosec} \theta = \frac{m^2+1}{2m}$ and $m > 0$ ( $0 < \theta < \frac{\pi}{2}$ ), find the value of the remaining trigonometric ratios.
50) If $\cot \theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quad., find the value of $\frac{3\sin \theta + 4\cos \theta}{\cos \theta - \sin \theta}$ .
51) Prove the following identities, state the domain of $\theta$ in each case:
52) $\sin^3 \theta - \cos^3 \theta = (\sin \theta - \cos \theta)(1 + \sin \theta \cos \theta)$
53) Prove the following identities, state the domain of $\theta$ in each case:
54) $\sin^6 \theta - \cos^6 \theta = (\sin^2 \theta - \cos^2 \theta)(1 + \sin^2 \theta \cos^2 \theta)$
55) Prove that: $\frac{\sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{\cot^2(\frac{3\pi}{2}-\theta)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)} = \cos \theta$
56) Show that $\cos(\alpha + B) + \sin(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta = \cos^2 \beta - \sin^2 \alpha$
57) Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$ , given that: $\tan \alpha = \frac{3}{4}$ , $\cos \beta = \frac{5}{13}$ and neither the terminal side of the angle of measure $\alpha$ nor that of $\beta$ is in the I quadrant.
58) Prove the identities: $\sqrt{\frac{1+\sin \alpha}{1-\sin \alpha}} = \frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$

59) Prove the identities:  $\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta} = 4\cos 2\theta$

60) Prove the identities:  $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

61) Prove the identities:  $\frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta} = \tan \frac{\alpha - \beta}{2} \cot \frac{\alpha + \beta}{2}$

## LONG Q.NO. 9

62) Solve the right triangle ABC, in which  $\gamma = 90^\circ$ ,  $b = 68.4$ ,  $c = 96.2$

63) Solve the triangle ABC, if  $\beta = 60^\circ$ ,  $\gamma = 15^\circ$ ,  $b = \sqrt{6}$

64) Solve the triangle ABC in which:  $b = 95$ ,  $c = 34$  and  $\alpha = 52^\circ$

65) Solve the triangles, using first Law of tangents and then Law of sines:  $b = 14.8$ ,  $c = 16.1$  and  $\alpha = 42^\circ 45'$

66) Solve the triangles, using first Law of tangents and then Law of sines:  $b = 61$ ,  $a = 32$  and  $\alpha = 59^\circ 30'$

67) The sides of a triangle are  $x^2 + x + 1$ ,  $2x + 1$  and  $x^2 - 1$ . Prove that the greatest angle of the triangle is  $120^\circ$

x) Show that: i)  $r = 4R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$  ii)  $s = 4R \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2}$

68) Find  $R$ ,  $r$ ,  $r_1$ ,  $r_2$  and  $r_3$ , if measures of the sides of triangle ABC are  $a = 13$ ,  $b = 14$ ,  $c = 15$

69) Prove that:  $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{9}{19}$

70) Prove that:  $3\tan^{-1} \frac{2}{3} = \sin^{-1} \frac{12}{13}$

71) Prove that:  $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} = \sin^{-1} \frac{77}{85}$

72) Prove that:  $\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6} = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$

73) Prove that:  $2\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$

74) Prove that:  $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$

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